ECE 307

Homework 3 Solutions

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CLONE MANUFACTURING COMPANY

- **Notation:**

N **manufacturers** : $j = 1, 2, \ldots, N$

M **plants** : $i = 1, 2, \ldots, M$

D **classes of boards** : $k = 1, 2, \ldots, D$

plant i **requires** R_{ik} **boards** $i = 1, \ldots, M$

$k = 1, \ldots, D$

CLONE MANUFACTURING COMPANY

x_j = number of boards from manufacturer j

c_j = costs per board from manufacturer j

U_j = maximum number of boards from manufacturer j

p_{jk} = fraction of class k boards from manufacturer j

c_{ji} = costs of shipping per board from manufacturer j to plant i

$j = 1, \ldots, N \qquad i = 1, \ldots, M \qquad k = 1, \ldots, D$

CLONE MANUFACTURING COMPANY

- **Observations:**

$$p_{jk} \geq 0 \quad \text{and} \quad \sum_{k=1}^{D} p_{jk} = 1 \qquad j = 1, \ldots, N$$

- **Decision variables:**

x_j = number of boards from manufacturer j

x_{ji} = number of boards shipped from manufacturer j to plant i

- **Objective:**

$$\min \sum_{j=1}^{N} c_j x_j + \sum_{j=1}^{N} \sum_{i=1}^{M} c_{ji} x_{ji}$$

CLONE MANUFACTURING COMPANY

- **Constraints:**

$$\sum_{j=1}^{N} p_{jk}\, x_{ji} = R_{ik} \qquad k = 1, 2, \ldots, D\ ,\ i = 1, \ldots, M$$

$$x_j \leq U_j \qquad j = 1, 2, \ldots, N$$

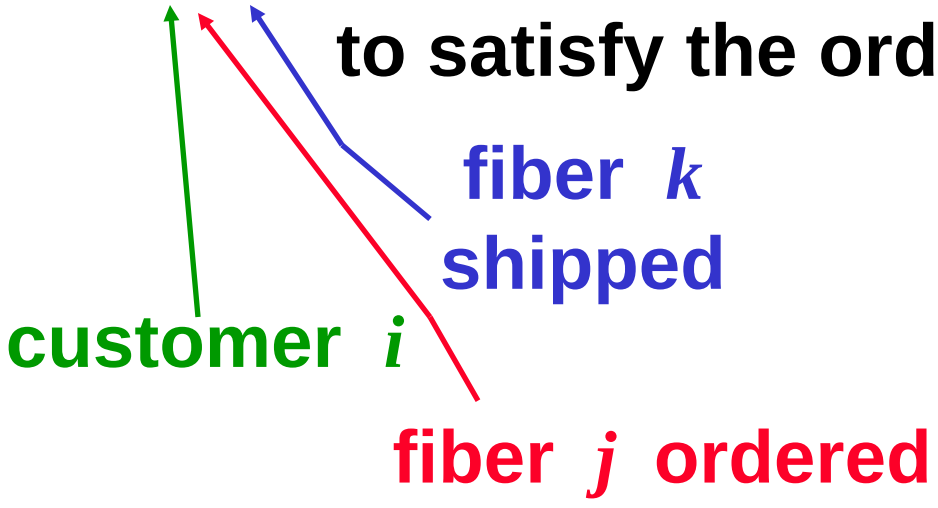
$$\sum_{i=1}^{M} x_{ji} \leq x_j \qquad j = 1, 2, \ldots, N$$

$$x_j \geq 0 \qquad j = 1, 2, \ldots, N$$

$$x_{ji} \geq 0 \qquad j = 1, 2, \ldots, N \quad i = 1, 2, \ldots, M$$

FAYE STOUT COMPANY : NOTATION

x_{ijk} = quantity of fiber k shipped to customer i to satisfy the order q_{ij} for fiber j



$k = j$
product demanded is the product shipped

$k \neq j$
a substitute product is shipped

$j = 1, \ldots, F$ $\quad F$ = number of fiber types

$k = 1, \ldots, F$

$i = 1, \ldots, C$ $\quad C$ = number of customers

FAYE STOUT COMPANY : NOTATION

q_{ij} = quantity of fiber j demanded by the customer i

A_j = quantity of fiber j available for shipment

c_{ijk} = costs per unit of shipping fiber k to customer i who ordered fiber j and the term may include a penalty for substitution

Note : whenever substitution is not allowed, such a penalty is made very large

FAYE STOUT COMPANY : NOTATION

x_j = fraction of each customer's fiber j order that is met with fiber j and permitted substitutes; x_j is uniform for all customers i

d_{ij} = penalty per unit of fiber j ordered by customer i but not filled with fiber j or the permitted substitutes

FAYE STOUT COMPANY : INFORMATION PROVIDED

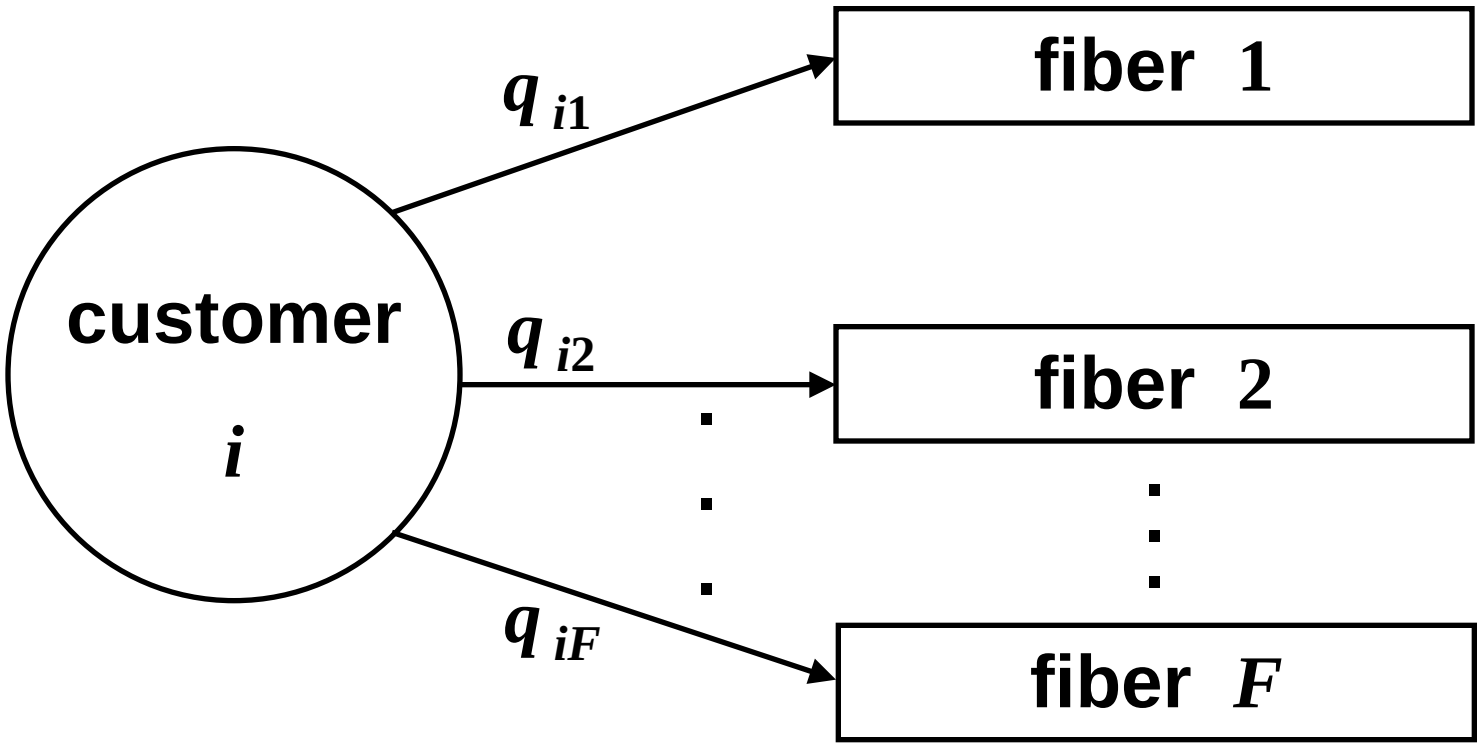
Φ_j = fair share for fiber j

quantity of fiber received

$.95\,\Phi_j \leq$ by each customer i of $\leq 1.05\,\Phi_j$

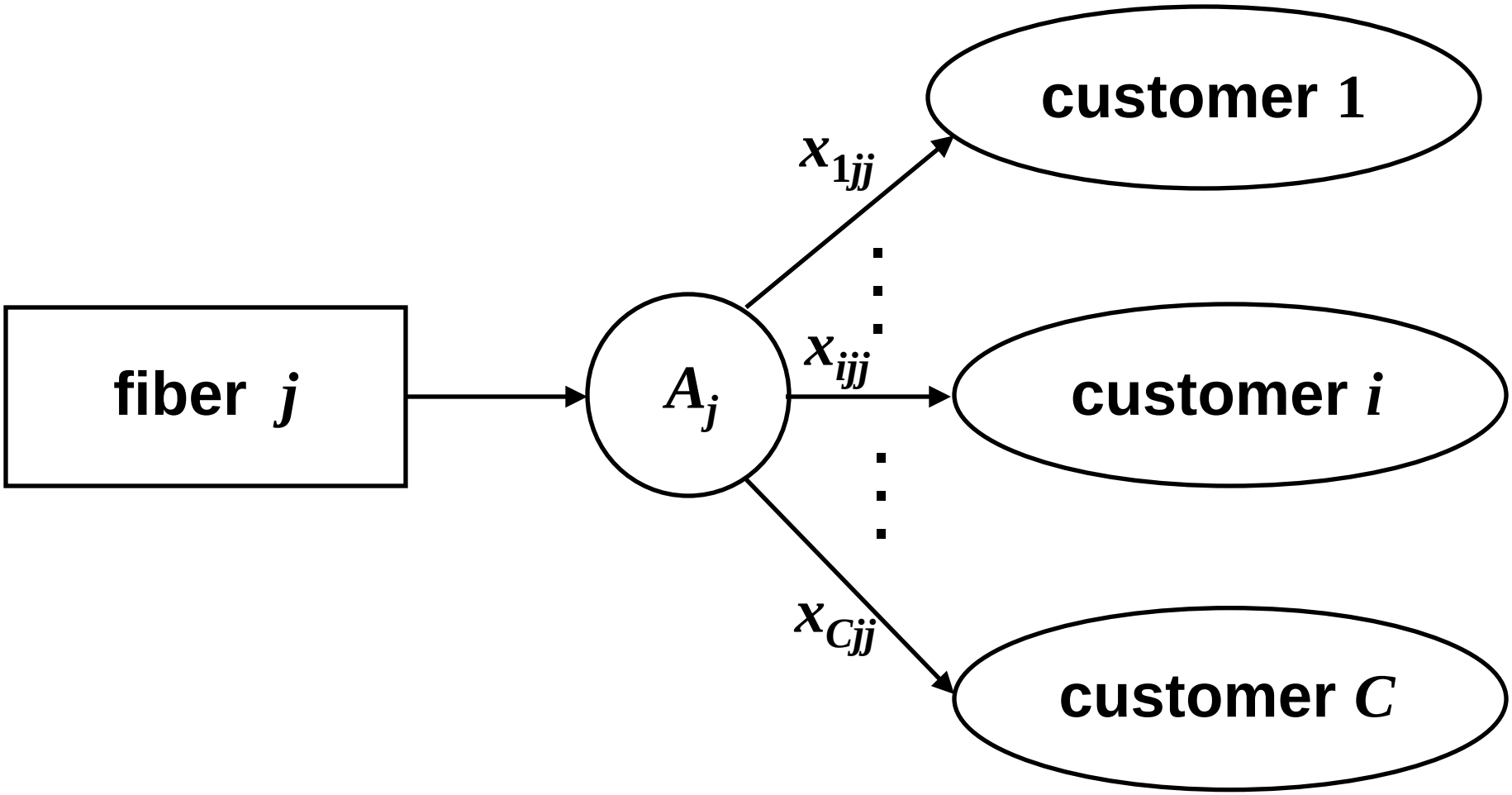
fiber j in short supply

FAYE STOUT COMPANY : FLOWS



q_{ij} are fixed and known data – parameters

FAYE STOUT COMPANY : FLOWS



fiber j delivery to customers

FAYE STOUT COMPANY : FLOWS

availability of fiber j is A_j ; however, demand is

$$\sum_{i=1}^{C} q_{ij} \;=\; Q_j \leftarrow \text{total demand for fiber } j \left[\text{ fixed}\right]$$

fair share is defined by

$$\Phi_j \triangleq \frac{A_j}{Q_j} \leftarrow \text{ fixed parameter for } j = 1, 2, \ldots, F$$

fiber j is in short supply if and only if

$$\Phi_j \;<\; 1$$

FAYE STOUT COMPANY : DECISION VARIABLES

x_{ijk} = amount of fiber k sent to meet customer i's demand for fiber j

y_{ij} = amount of fiber j not supplied to customer i, or more precisely, amount of fiber j ordered by customer i but not filled with either fiber j or the permitted substitutes

FAYE STOUT COMPANY : OBJECTIVE

$$\min \quad \underbrace{\sum_{i=1}^{C}\sum_{j=1}^{F}\sum_{k=1}^{F} c_{ijk}\, x_{ijk}}_{\text{costs of items supplied}} \quad + \quad \underbrace{\sum_{i=1}^{C}\sum_{j=1}^{F} d_{ij}\, y_{ij}}_{\text{penalties incurred for items not supplied}}$$

FAYE STOUT COMPANY : CONSTRAINTS

- balance

$$\sum_{k=1}^{F} x_{ijk} + y_{ij} = q_{ij} \quad \begin{matrix} i=1,\ldots,C \\ j=1,\ldots,F \end{matrix}$$

- availability

$$\sum_{i=1}^{C} x_{ijj} \leq A_j \quad j=1,\ldots,F$$

- uniform fraction of order filled for fiber j

$$\frac{1}{q_{ij}} \sum_{k=1}^{F} x_{ijk} = x_j \quad i=1,2,\ldots,C$$

FAYE STOUT COMPANY : CONSTRAINTS

- fair share constraints

$$j = 1, 2, \ldots, F$$

$$0.95\,\Phi_j \leq x_j \leq 1.05\,\Phi_j$$

such that $\Phi_j < 1$

- nonnegativity

$$x_{ijk} \geq 0 \qquad \forall\, i, \forall\, j, \forall\, k$$

$$y_{ij} \geq 0 \qquad \forall\, i, \forall\, j$$

THE MONTY ZOOMA COMPANY

- **Problem data:**
  - **18 – month production schedule**
  - **each worker produces 300 bottles per month**
  - **stored items from month t to month $t + 1$ incur a 5 % loss**
  - **$n_0 = 50$ workers for initial month**
  - **each month t**
    - **new workers hired**
    - **old workers released**
    - **workers kept idle**

THE MONTY ZOOMA COMPANY

- attrition rates for workers are

  10 % for those who are idled

  1 % for those who are in production

- Decision variables are associated with costs

$c_t \leftrightarrow e_t$ = number of workers in production

$h_t \leftrightarrow x_t$ = number of workers hired

$f_t \leftrightarrow y_t$ = number of workers released

$n_t \leftrightarrow d_t$ = number of workers idle

decisions at the beginning of each month t

THE MONTY ZOOMA COMPANY

month $t = 1, 2, \ldots, 18$

$i_t \leftrightarrow s_t$ = bottles in storage at the end of the month t

S_t = number of bottles sold in month t

- Terminal constraints are given by

$$s_{18} \geq I / 0.95$$

work force at $t = 19 \geq W$

THE MONTY ZOOMA COMPANY

- The objective is to minimize the costs of production
  - we ignore costs of resources other than labor for period t and so costs are employment plus storage for each month t

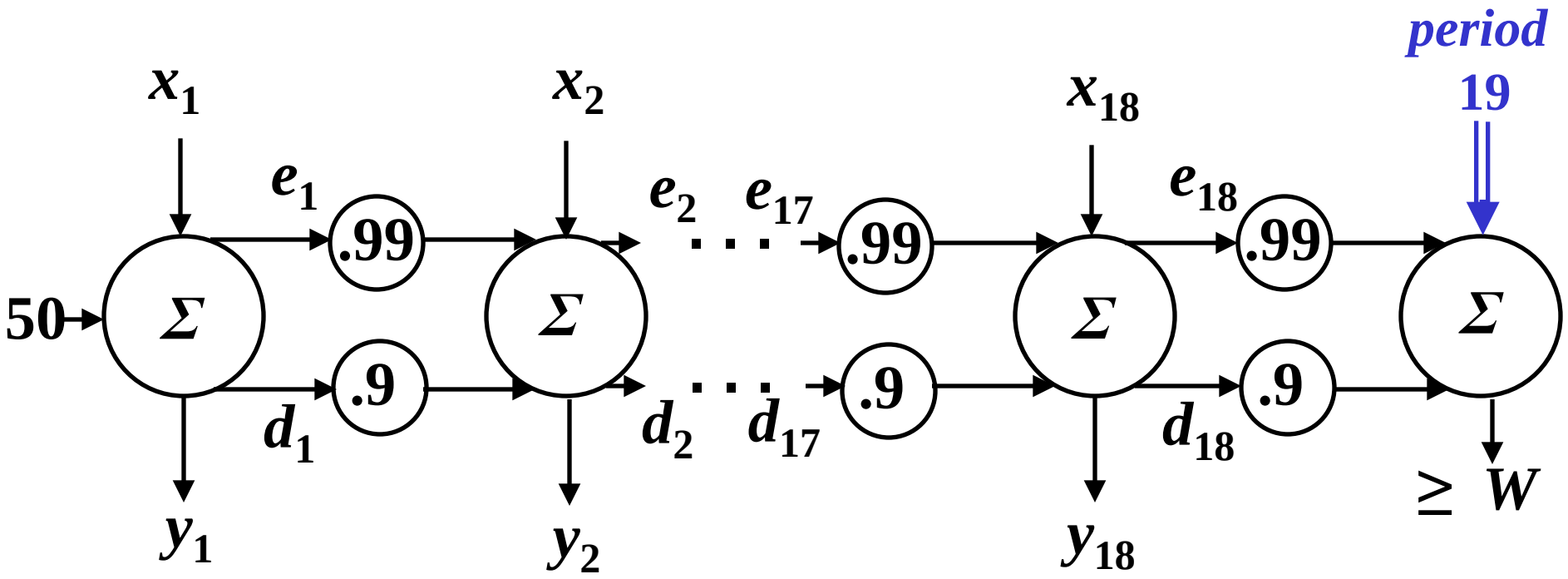
$$c_t e_t + h_t x_t + f_t y_t + n_t d_t + i_t s_t$$

  - the objective is

$$min \sum_{t=1}^{18} \left[c_t e_t + h_t x_t + f_t y_t + n_t d_t + i_t s_t \right]$$

THE MONTY ZOOMA COMPANY : CONSTRAINTS

- work-force constraints:



period 1

$$50 + x_1 - y_1 = e_1 + d_1$$

$$.99e_1 + .9d_1 + x_2 - y_2 = e_2 + d_2$$

THE MONTY ZOOMA CORPORATION

general relationship

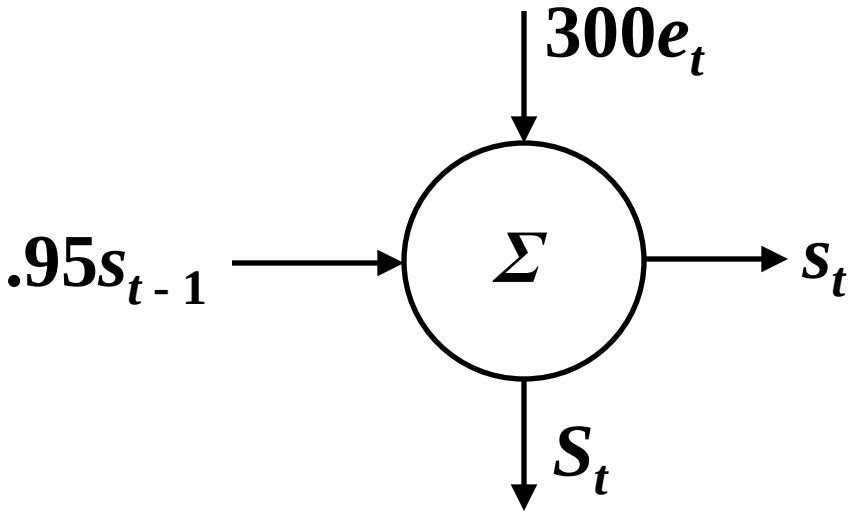
$$.99e_{t-1} + .9d_{t-1} + x_t - y_t = e_t + d_t \qquad t = 2, \ldots, 18$$

terminal requirement

$$.99e_{18} + .9d_{18} \geq W$$

THE MONTY ZOOMA CORPORATION : CONSTRAINTS

- production levels



general relationship

$$300\, e_t = S_t + s_t - .95\, s_{t-1} \qquad t = 1, \ldots, 18$$

terminal requirements

$$s_0 = 0 \qquad .95\, s_{18} \geq I$$

THE MONTY ZOOMA CORPORATION : PROBLEM STATEMENT

$$\min \sum_{t=1}^{18} \left\{ c_t e_t + h_t x_t + f_t y_t + n_t d_t + i_t s_t \right\}$$

$$e_1 + d_1 - x_1 + y_1 = 50$$

$$.99e_{t-1} + .9d_{t-1} + x_t - y_t - e_t - d_t = 0 \quad t = 2, \ldots, 18$$

$$.99e_{18} + .9d_{18} \geq W$$

$$300e_1 - s_1 = S_1$$

$$300e_t - s_t + 0.95s_{t-1} = S_t \quad t = 2, \ldots, 18$$

$$0.95s_{18} \geq I$$

$$e_t, x_t, y_t, d_t, s_t, \geq 0$$